# REVIEW 3

(1) Informally show that there are infinitely many Turing machines which not only diverge on infinitely many inputs, but also halt for infinitely many inputs.
(2) Informally show that there are infinitely many Turing machines which diverge on infinitely many inputs, but halt for finitely many inputs.
(3) Informally show why the number of total functions is greater than the number of Turing machines.
(4) Suppose that the sets *A, B,* and their complements *A_, B_,* are recursively enumerable (r.e.). Is the set *C* = *A_ _B* recursive ? Present a formal proof.
(5) Suppose that the set *D* is only r.e. Is the set *E* = *D_D_* recursive? Present a formal proof.
(6) Show that for any positive integer *k,* the line segment (1/2*k,* 1/*k*) contains uncountably many irrational numbers.
(7) Informally prove the statement: *A Turing machine cannot verify the divergence of another Turing machine*.
(8) Is the number of total boolean functions greater than the number of recursive functions? Justify your answer.
(9) Is the cardinality of the set of partially recursive functions greater than the cardinality of the set of recursive functions? Justify your answer.
(10) Can a set and its complement BOTH be non-r.e.? Can a set and its complement BOTH be r.e.? Explain.
(11) Is $L = \{ i$ such that $M_i$ accepts only one string $\}$ an r.e. set?
(12) Show that the set *K* has an undecidable membership problem.
(13) How would one go about proving that a given set is not computable.
(14) Is following a regular set $\{1^p$ where $p$ is a prime and $p > 10000\}$? Prove your answer.
(15) Show that every infinite regular set has an infinite regular subset.
(16) Justify the claim that the question *Does a PDA accept everything?* is undecidable.
(17) Show that every finite regular set has an infinite regular complement.
(18) Is the language consisting of binary strings *x* such that *x* MOD *x* = 0 regular? Prove your answer.
(19) Show that every infinite regular set has a finite regular subset.
(20) Give a regular expression for odd-length strings (over the alphabet $\{0,1\}$) that contain 11 as a prefix or as a suffix (or both).
(21) Do there exist finite sets that can be recognized by PDA but not by FSA? Prove your answer.
(22) Are regular, CFL, CSL, and r.e. sets closed under union? Are regular, CFL, CSL, and r.e. sets closed under concatenation? Are regular, CFL, CSL, and r.e. sets closed under intersection? Prove your answers.
(23) Can ambiguity always be eliminated from Context Free Grammars?
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(24) Is it true that every CFL is also a CSL?
(25) Is it true that every CFL is also regular?
(26) Is the language $1^{n^2}$ regular? CFL? Prove your answers.
(27) Change the grammar below to Chomsky's Normal Form.

S → CB
C → aCb
C → ab
B → cB
B → c

(28) What is a direct implication of the G.N.F. for C.F.Gs?
(29) Discuss the closure of r.e., recursive, CSL, CFL, and Regular sets under the operation of Complementation.
(30) Describe a problem which you think is harder than the Halting problem.
(31) Is $L = \{ < i,j > \text{ s.t. } M_i \text{ and } M_j \text{ accept different sets} \}$ an r.e. set? Prove your answer.
(32) Is the problem of determining if a Turing machine $M_i$ will execute exactly fifteen steps on input 99 solvable? (Justify).
(33) Is the membership problem for an intersection of finitely many infinite sets solvable? (Prove).

(34) Is the set of indices of Turing machines that accept at least *k* inputs an r.e. set?
(Prove).
(35) Discuss the concept of ambiguity in *regular expressions*. What does it mean? Is it desirable? Can it always be eliminated?
(36) What is harder to detect: the *Union* or *Concatenation* ambiguity? (Explain).